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Electron-ion Coulomb Bremsstrahlung in nonideal plasmas

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Abstract. The classical electron-ion Coulomb Bremsstrahlung process is investigated in nonideal plasmas. An effective pseudopotential model taking into account the plasma screening and collective effects is applied to describe the electron-ion interaction potential in a classical nonideal plasma. The classical straight-line trajectory method is applied to the motion of the projectile electron in order to visualize the variation of the differential Bremsstrahlung radiation cross-section (DBRCS) as a function of the scaled impact parameter, nonideal plasma parameter, projectile energy, photon energy, and Debye length. The results show that the DBRCS in ideal plasmas described by the Debye-Hückel potential is always greater than that in nonideal plasmas, *i.e.*, the collective effects reduce the DBRCS for both the soft and hard photon cases. For large impact parameters, the DBRCS for the soft photon case is found to be always greater than that for the hard photon case.

PACS. 52.20.-j Elementary processes in plasma

1 Introduction

Bremsstrahlung emission [1–11] in plasmas has received much attention since these processes have been widely used for plasma diagnostics in laboratory and astrophysical plasmas. Recently, Bremsstrahlung processes in weakly coupled plasmas [7,10] have been investigated using the Debye-Hückel model [12] potential with the classical trajectory method. This Debye-Hückel effective potential describes the properties of a low density plasma and corresponds to a pair correlation approximation. The plasmas described by the Debye-Hückel model are called ideal plasmas since the average energy of interaction between particles is small compared to the average kinetic energy of a particle [13]. However, multiparticle correlation effects caused by the simultaneous interaction of many particles with increasing the plasma density should be taken into account. It is necessary to take into account not only short-range collective effects but also, in the case of a plasma with a moderate density and temperature, long range effects. In this case, the interaction potential is different from the Debye-Hückel type because of the strong collective effects of nonideal particle interaction [14]. Then, the Bremsstrahlung radiation spectrum due to electron-ion Coulomb scattering in nonideal plasmas is different from that in ideal plasmas. In this paper we investigate Bremsstrahlung processes in electron-ion Coulomb scattering in nonideal plasmas. A pseudopotential model including the plasma screening effects and collective effects is applied to describe the interaction potential between the projectile electron and target ion in nonideal plasmas. The classical straight-line (SL) trajectory method [2,3,15,16] is applied to obtain the differential Bremsstrahlung radiation cross section (DBRCS) as a function of the scaled impact parameter, nonideal plasma parameter, projectile energy, photon energy, and Debye length.

In Section 2, we discuss the classical expression of the DBRCS in Coulomb scattering of nonrelativistic projectile electron with target ion in nonideal plasmas described by the pseudopotential model. In Section 3 we obtain the closed form of the DBRCS using the components of the force parallel ($F_{\|}$) and perpendicular ($F_{\perp}$) to the velocity of the projectile electron. We also investigate the variation of the DBRCS for both the soft and hard photon radiation cases with a change of the impact parameter. The results show that the DBRCS in ideal plasmas described by the Debye-Hückel potential is always greater than in nonideal plasmas described by the pseudopotential. For large impact parameters, the DBRCS for the soft photon radiation is found to be always greater than that for the hard photon radiation. Finally, in Section 4, a discussion is given.

2 Classical DBRCS

The classical expression of the DBRCS [3] is given by

$$\mathrm{d}\sigma_b = 2\pi \int \mathrm{d}b\, b\, \mathrm{d}w_\omega(b), \qquad (1)$$

where b is the impact parameter and $\mathrm{d}w_\omega$ is the differential probability of emitting a photon of frequency ω within $\mathrm{d}\omega$ when a projectile particle changes its velocity in collisions

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with a static target system. For all impact parameters, $\mathrm{d}w_\omega$ is given by the Larmor formula [12] for the emission spectrum of a nonrelativistic accelerated charge:

$$\mathrm{d}w_\omega = \frac{8\pi e^2}{3\hbar c^3} |\mathbf{a}_\omega|^2 \frac{\mathrm{d}\omega}{\omega}, \tag{2}$$

where $\mathbf{a}_\omega$ is the Fourier coefficient of the acceleration $\mathbf{a}(t)$ of the projectile electron

$$\mathbf{a}_\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}t \, \mathrm{e}^{\mathrm{i}\omega t} \mathbf{a}(t). \tag{3}$$

In the SL trajectory method, the position of the projectile electron can be represented as $\mathbf{r}(t) = \mathbf{b} + \mathbf{v}t$ with $\mathbf{b} \cdot \mathbf{v} = 0$, where $\mathbf{v}$ is the velocity of the projectile ion. To compute $\mathbf{a}_\omega$ we can set up coordinate axes so that the electron orbit is in the collision plane; then

$$|\mathbf{a}_\omega|^2 = \frac{1}{m^2} \left(|F_{\|\omega}|^2 + |F_{\perp\omega}|^2 \right), \tag{4}$$

where $F_{\|\omega}$ and $F_{\perp\omega}$ are the Fourier coefficients components of force $\mathbf{F}(t)$ parallel and perpendicular to the projectile velocity, respectively,

$$F_{\mu\omega} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathrm{d}t \, \mathrm{e}^{\mathrm{i}\omega t} F_\mu, \quad (\mu = \|, \perp). \tag{5}$$

The non-straight-line character of the motion of the projectile electron in the field of the scattering potential can also be investigated using the modified hyperbolic-orbit curved trajectory method [9,17] including the plasma screening effect. Since the investigation of the nonideal effects on the electron-ion Bremsstrahlung processes in nonideal plasmas is the main purpose of this work, the SL trajectory method has been used throughout this paper. It has been known that the DBRCSs using the SL trajectory method and the curved trajectory method are almost identical for large impact parameters $b > a_Z$, where a_Z $(\equiv a_0/Z = \hbar^2/Zme^2)$ is the first Bohr radius of a hydrogenic ion with nuclear charge Z and m is the electron rest mass. Thus, the SL trajectory method for the large impact parameter region is quite reliable.

Recently, an integro-differential equation for the effective potential of the particle interaction taking into account the simultaneous correlations of N particles was obtained on the basis of a sequential solution of the chain of Bogolyubov equations for the equilibrium distribution function of particles of a classical nonideal plasma [14]. Also, in a recent paper [14], an analytic expression for the pseudopotential of the particle interaction in nonideal plasmas was obtained by application of the spline-approximation. Using the pseudopotential taking into account the plasma screening effects and collective effects, the interaction potential $V_{\mathrm{NI}}(\mathbf{r})$ between the projectile electron and the target ion with charge Z in nonideal plasmas can be represented by

$$V_{\mathrm{NI}}(\mathbf{r}) = -\frac{Ze^2}{r} \mathrm{e}^{-r/\Lambda} \frac{1 + \gamma f(r)/2}{1 + c(\gamma)}, \tag{6}$$

where $\mathbf{r}$ is the position vector of the projectile electron from the target ion, Λ is the Debye length,

$$f(r) = (\mathrm{e}^{-\sqrt{\gamma} r/\Lambda} - 1)(1 - \mathrm{e}^{-2r/\Lambda})/5,$$

and γ $(\equiv e^2/\Lambda k T_\mathrm{e})$ is the nonideal plasma parameter,

$$c(\gamma) \cong -0.008617 + 0.455861\gamma - 0.108389\gamma^2 + 0.009377\gamma^3$$

is the correlation coefficient for different values of γ, and T_e is the electron temperature. When $\gamma \ll 1$, *i.e.*, for weakly nonideal or rare ideal plasmas, the pseudopotential (Eq. (6)) goes over into the Debye-Hückel potential $V_{\mathrm{DH}}(\mathbf{r}) = -(Ze^2/r)\mathrm{e}^{-r/\Lambda}$. The use of the pseudopotential (Eq. (6)) and the SL trajectory impact parameter approach allows us to derive analytic formulas for the Fourier coefficients of the force:

$$F_{\mu\omega} = -\frac{Ze^2}{\pi \bar{v} a_Z} \frac{1}{1 + c(\gamma)} \bar{F}_{\mu\omega}, \quad (\mu = \|, \perp), \tag{7}$$

where $\bar{v} \equiv v/a_Z$ and the scaled Fourier coefficients $\bar{F}_{\|\omega}$ and $\bar{F}_{\perp\omega}$ are given by

$$\bar{F}_{\|\omega} = \mathrm{i} \int_0^\infty \mathrm{d}\tau \, \tau \sin \xi\tau \times \left[\frac{1 + a_\Lambda}{\bar{r}^3} J(\bar{r}, \gamma, a_\Lambda) - \frac{a_\Lambda}{\bar{r}^2} K(\bar{r}, \gamma, a_\Lambda) \right], \tag{8}$$

$$\bar{F}_{\perp\omega} = \int_0^\infty \mathrm{d}\tau \, \bar{b} \cos \xi\tau \times \left[\frac{1 + a_\Lambda}{\bar{r}^3} J(\bar{r}, \gamma, a_\Lambda) - \frac{a_\Lambda}{\bar{r}^2} K(\bar{r}, \gamma, a_\Lambda) \right], \tag{9}$$

where τ $(\equiv \bar{v}t)$ is the scaled time, $\xi \equiv \omega/\bar{v}$, a_Λ $(\equiv a_Z/\Lambda)$ is the reciprocal scaled Debye length, $\bar{r}$ $(\equiv r/a_Z) = \sqrt{\tau^2 + \bar{b}^2}$, and $\bar{b}$ $(\equiv b/a_Z)$ is the scaled impact parameter. Here the functions $J(\bar{r}, \gamma, a_\Lambda)$ and $K(\bar{r}, \gamma, a_\Lambda)$ are given by, respectively,

$$J(\bar{r}, \gamma, a_\Lambda) = \mathrm{e}^{-a_\Lambda \bar{r}} + \frac{\gamma}{10} \Big[-\mathrm{e}^{-(\sqrt{\gamma}+3) a_\Lambda \bar{r}} + \mathrm{e}^{-(\sqrt{\gamma}+1) a_\Lambda \bar{r}} + \mathrm{e}^{-3 a_\Lambda \bar{r}} - \mathrm{e}^{-a_\Lambda \bar{r}} \Big], \tag{10}$$

$$K(\bar{r}, \gamma, a_\Lambda) = \frac{\gamma}{10} \Big[(\sqrt{\gamma} + 2) \mathrm{e}^{-(\sqrt{\gamma}+3) a_\Lambda \bar{r}} - \sqrt{\gamma} \mathrm{e}^{-(\sqrt{\gamma}+1) a_\Lambda \bar{r}} - 2\mathrm{e}^{-3 a_\Lambda \bar{r}} \Big]. \tag{11}$$

Then, in the nonrelativistic limit, the classical DBRCS is found to be

$$\mathrm{d}\sigma_b = \frac{16}{3} \frac{\alpha^3 a_0^2}{\bar{E}} \frac{\mathrm{d}\omega}{\omega} \int_{\bar{b}_{\min}}^{\bar{b}_{\max}} \mathrm{d}\bar{b} \, \bar{b} \left(|\bar{F}_{\|\omega}|^2 + |\bar{F}_{\perp\omega}|^2 \right), \tag{12}$$

where α $(= e^2/\hbar c \cong 1/137)$ is the fine structure constant, $\bar{E}$ $(\equiv mv^2/2Z^2 Ry)$ is the scaled kinetic energy of

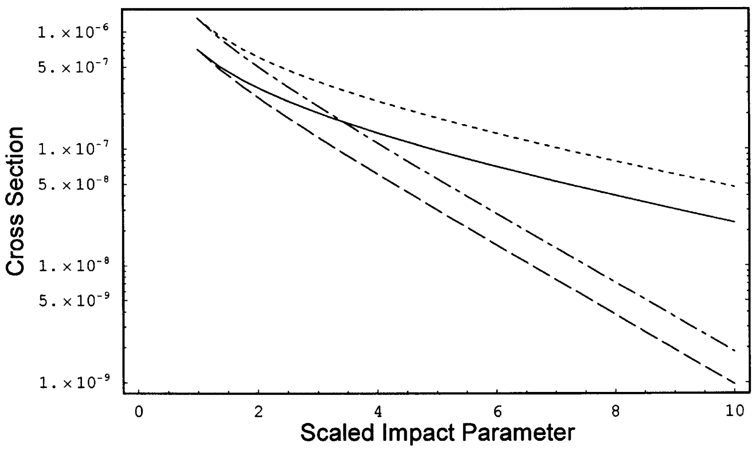

Fig. 1. The scaled double differential Bremsstrahlung cross-section ($\mathrm{d}^2\chi_b/\mathrm{d}\bar{\varepsilon}\mathrm{d}\bar{b}$) in units of πa_0^2 as a function of the scaled impact parameter ($\bar{b} = b/a_Z$) when $\bar{E} = 0.5$. Solid line: soft photon radiation $\bar{\varepsilon}/\bar{E} = 0.1$ in nonideal plasmas with $\gamma = 1$. Dotted line: soft photon radiation $\bar{\varepsilon}/\bar{E} = 0.1$ in ideal plasmas, *i.e.*, $\gamma = 0$. Dashed line: hard photon radiation $\bar{\varepsilon}/\bar{E} = 0.9$ in nonideal plasmas with $\gamma = 1$. Dot-dashed line: hard photon radiation $\bar{\varepsilon}/\bar{E} = 0.9$ in ideal plasmas, *i.e.*, $\gamma = 0$.

the projectile electron, and Ry ($= me^4/2\hbar^2 \cong 13.6$ eV) is the Rydberg constant. Here the scaled minimum impact parameter $\bar{b}_{\min}$ corresponds to the closest distance of approach at which the electrostatic potential energy of interaction is equal to the maximum possible energy transfer [6], *i.e.*

$$2mv^2 = \frac{Ze^2}{b}\mathrm{e}^{-b/\Lambda}\frac{1+\gamma f(b)/2}{1+c(\gamma)}\,. \tag{13}$$

The scaled maximum impact parameter $\bar{b}_{\max}$ is determined by the screening length for the Coulomb forces, *i.e.*, the Debye length: $\bar{b}_{\max} \approx a_\Lambda^{-1}$.

3 Determination of the modified DBRCS

The DBRCS [12] is defined by

$$\frac{\mathrm{d}\chi_b}{\mathrm{d}\varepsilon} \equiv \frac{\mathrm{d}\sigma_b}{\hbar\mathrm{d}\omega}\hbar\omega, \tag{14}$$

$$= \frac{16}{3}\frac{\alpha^3 a_0^2}{\bar{E}}\int\limits_{\bar{b}_{\min}}^{\bar{b}_{\max}} \mathrm{d}\bar{b}\,\bar{b}\left(\left|\bar{F}_{\|\omega}\right|^2 + \left|\bar{F}_{\perp\omega}\right|^2\right), \tag{15}$$

where ε ($\equiv \hbar\omega$) is the radiation photon energy. In nonrelativistic limit, the parameter ξ can be rewritten as $\xi = \bar{\varepsilon}/2\sqrt{\bar{E}}$, where $\bar{\varepsilon}$ ($\equiv \hbar\omega/Z^2Ry$) is the scaled photon energy. Then, the scaled double DBRCS, *i.e.*, the scaled DBRCS per scaled impact parameter, can be presented as

$$\frac{\mathrm{d}^2\chi_b}{\mathrm{d}\bar{\varepsilon}\mathrm{d}\bar{b}}/\pi a_0^2 = \frac{16}{3\pi}\frac{\alpha^3}{\bar{E}}\left[1+c(\gamma)\right]^{-2}\bar{b} \times\left[\left|\bar{F}_{\|\omega}\left(\bar{b},\gamma,a_\Lambda,\bar{\varepsilon},\bar{E}\right)\right|^2 + \left|\bar{F}_{\perp\omega}\left(\bar{b},\gamma,a_\Lambda,\bar{\varepsilon},\bar{E}\right)\right|^2\right]. \tag{16}$$

In order to investigate the plasma screening effects on the DBRCS, we consider two cases for the ratio of the radiation photon energy to the kinetic energy of the projectile electron: $\bar{\varepsilon}/\bar{E}$ ($= 2\hbar\omega/mv^2$) $= 0.1$ (soft photon) and 0.9 (hard photon). Here we choose $a_\Lambda = 0.1$, $\gamma = 1$, and $\bar{E} = 0.5$ since the classical trajectory approximation is known to be reliable for low energy projectiles ($v < Z\alpha c$) [3]. Figure 1 shows the scaled double DBRCS (Eq. (16)) for electron-ion Coulomb collisions in ideal and nonideal plasmas for large impact parameters $b > a_Z$. As we can see in this figure, the DBRCS in ideal plasmas ($\gamma = 0$), as described by the Debye-Hückel potential, is found to be always greater than in nonideal plasmas ($\gamma = 1$) described by the pseudopotential. Thus the nonideality ($\gamma \neq 0$) of plasmas, *i.e.*, the collective effect, reduces the DBRCS for both the soft and hard photon cases. For large impact parameters ($\bar{b} > 1$), the DBRCS for the soft photon case is always greater than that for the hard photon case.

4 Discussion

We have investigated the plasma screening effects and collective effects on the Coulomb Bremsstrahlung in electron-ion scattering in classical nonideal plasmas. An effective pseudopotential model has been applied to describe the electron-ion interaction in nonideal plasmas. Further, the classical straight-line trajectory method has been applied to the motion of the projectile electron in order to visualize the variation of the DBRCS as a function of the scaled impact parameter ($\bar{b}$), nonideal plasma parameter (γ), projectile energy (E), photon energy (ε), and Debye length (Λ). The results show that the DBRCS in ideal plasmas, as described by the Debye-Hückel model, is always greater than that in nonideal plasmas, *i.e.*, the collective effect ($\gamma \neq 0$) suppresses the DBRCS for both the soft and hard photon radiation cases. For large impact parameters, the DBRCS for the soft photon radiation case is found to be always greater than that for the hard photon radiation case. These results provide useful information on the Bremsstrahlung processes including the plasma screening effects and collective effects in electron-ion scattering in nonideal plasmas.

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